

## MATHEMATICAL LOGIC — ASSIGNMENT THREE

- (1) Define ordinal multiplication.

*This is Definition 16.16 in the slides.*

- (2) Prove that there is no enumeration  $\{f_x\}_{x \in \mathbb{N}}$  of all total computable functions which admits a computable enumeration function  $e(x, z) = f_x(z)$ .

*This is Theorem 19.20 in the slides.*

- (3) Show that  $\lambda x. K x$  is strongly normalisable, but there are two  $\lambda$ -terms  $A$  and  $B$  such that  $(\lambda x. K x) A B$  is not strongly normalisable.

*Calculating;  $\lambda x. K x = \lambda x. (\lambda y, z. y) x \triangleright_\beta \lambda x, z. x = K$  which is irreducible, thus strongly normalisable. Since this is the only possible reduction, the original term is strongly normalisable.*

*Let  $A = \Omega$  and  $B = K$ . Then  $(\lambda x. K x) A B \triangleright_\beta K A B \triangleright_\beta A = \Omega$  which reduces to itself in one step, thus this reduction can be always extended. Hence  $(\lambda x. K x) A B$  is not strongly normalisable.*