

HISTORY OF MATHEMATICS — ASSIGNMENT TWO

- (1) (2 points) What is Hilbert’s “Entscheidungsproblem”?
It is the problem of finding an “effective procedure” (an algorithm) that could determine, for any given mathematical statement, whether it is true or false.
- (2) (4 points) Schönfinkel introduced the fundamental concept of combinators. Could you list three combinators?
S, K and I:
 - **S** $x y z = (x z)(y z)$;
 - **K** $x y = x$;
 - **I** $x = x$.
- (3) (3 points) Can you define λ -calculus’ β -contraction?
 β -contraction: $(\lambda x. M) N \triangleright M[x/N]$.
- (4) (4 points) What is the definition of a function defined by primitive recursion?
Assume g and h are partial recursive functions. Then f is defined by primitive recursion as
$$f(x_1, \dots, x_k, 0) = g(x_1, \dots, x_k) ;$$
$$f(x_1, \dots, x_k, n + 1) = h(x_1, \dots, x_k, n, f(x_1, \dots, x_k, n)) .$$
- (5) (2 points) What is a “universal Turing machine”?
A UTM is a single, fixed Turing machine that can simulate the behaviour of any other arbitrary Turing machine.
- (6) (3 points) Can you state Rice’s theorem?
Any non-trivial property of the language accepted (or function computed) by a Turing machine is undecidable.
- (7) (3 points) Who proved that Hilbert’s tenth problem is undecidable?
Yurii Matiyasevich (1970).
- (8) (1 points) Who wrote “First Draft of a Report on the EDVAC” (1945)?
John von Neumann.
- (9) (2 points) Why is ENIAC significant?
It is the first general-purpose electronic digital computer.
- (10) (4 points) Can you list the core components of a finite state automaton?
See slide 116.
- (11) (2 points) Who proved that the Boolean Satisfiability Problem is NP-complete in 1971?
Stephen Cook.
- (12) (2 points) What is the difference between bits and qubits?
Unlike classical bits (0 or 1), qubits can exist in multiple states simultaneously and be interconnected in complex ways.