MATHEMATICAL LOGIC — ASSIGNMENT TWO

(1) Prove $\vdash (\forall x.\, P \vee \neg P) \wedge \neg\forall x.\, \neg P \supset \exists x.\, P$.

(2) Prove the Downward Löwenheim-Skolem Theorem:

> Let T be a theory on the signature Σ with just one sort. If T has an infinite model of cardinality $\alpha \geq |\Sigma|$ then T has a model of cardinality $\max(|\Sigma|, \aleph_0)$.

(3) Show that for every partial order $\langle P; \leq_p \rangle$ there exists a total order $\langle P; \leq_T \rangle$ such that $\leq_P \subseteq \leq_T$.
(Hint: a set S is finite if and only if there is a function $f\colon \{n \mid 0 \leq n < |S|\} \to S$ bijective.)

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

Date: April 14^{th}, 2026.