MATHEMATICAL LOGIC — ASSIGNMENT FOUR

(1) Define the notion of representable relation in Peano arithmetic.
(2) Prove that in every Heyting algebra, for every x, y, z, $x \leq y \supset z$ if and only if $x \wedge y \leq z$.
(3) Assuming that Peano arithmetic is consistent, show that there is a sentence in Heyting arithmetic which is not provable and whose negation is not provable either.

    (Hint: use Gödel's First Incompleteness Theorem for Peano arithmetic. Beware that provability in Heyting arithmetic differs from provability in Peano arithmetic.)

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

Date: June 4^{th}, 2025.