MATHEMATICAL LOGIC — ASSIGNMENT THREE

(1) Prove that every infinite ordinal can be written as the ordinal sum of a limit ordinal and a finite ordinal.
(2) Show that there is a combinator $\mathbf{Y}$ such that $\mathbf{Y}x =_\beta x(\mathbf{Y}x)$.
(3) Let $\{\phi_i\}_{i\in\mathbb{N}}$ be a good enumeration of all the partial recursive functions. Let $A = \{j \colon \phi_j = \phi_{j+1}\}$. Is A recursive? Motivate your answer.
(Hint: Use Kleene's fixed point theorem.)

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

Date: May 21st, 2025.