MATHEMATICAL LOGIC — ASSIGNMENT TWO

(1) Prove $\vdash (\forall x.\, A) \vee B \supset \forall x.\, A \vee B$ where $x \notin \mathsf{FV}(B)$. Show a counterexample when $x \in \mathsf{FV}(B)$.

(2) State and prove the Compactness Theorem.

(3) In the definition of Σ-structure it is required that all sorts are interpreted in non-empty sets. Show that this requirement is necessary since the Soundness Theorem becomes invalid if the requirement is not met.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.