## MATHEMATICAL LOGIC — ASSIGNMENT TWO

(1) Prove $\vdash A[t/x] = \exists x.\, x = t \wedge A$ in natural deduction assuming $x \notin \mathsf{FV}(t)$.
(2) Show that, for any set of formulæ $\Gamma$ and any formula $A$,
   - $\Gamma \cup \{ \neg A \}$ is not consistent if and only if $\Gamma \vdash A$;
   - $\Gamma \cup \{ A \}$ is not consistent if and only if $\Gamma \vdash \neg A$.
(3) Show that every first-order theory $T$ on the signature

$$\langle S; \emptyset; \{ \leq : S \times S \} \rangle$$

having as models all the finite total orders, has necessarily an infinite model.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

*Date*: April $10^{\text{th}}$, 2024.