MATHEMATICAL LOGIC — ASSIGNMENT THREE

(1) Show that the collection $\mathbb{N}^*$ of finite sequences over the naturals is in bijective correspondence with $\mathbb{N}$.
    See Example 13.4 in the slides.
(2) State, without proving, the Zorn Lemma.
    This is Theorem 16.2 in the slides.
(3) In the pure λ-calculus, let $V \equiv \lambda y.x\,(y\,y)$ and let $R \equiv \lambda x.\,V\,V$. Show that, for all z, $R\,z =_\beta z\,(R\,z)$.
    Calculating:

$$\begin{aligned} & R\,z \\ \equiv\ & (\lambda x.\,V\,V)\,z \\ =_\beta\ & (V\,V)[z/x] \\ \equiv\ & V[z/x]\,V[z/x] \\ \equiv\ & (\lambda y.\,z\,(y\,y))\,V[z/x] \\ =_\beta\ & z\,(V[z/x]\,V[z/x]) \\ \equiv\ & z\,((V\,V)[z/x]) \\ =_\beta\ & z\,((\lambda x.\,V\,V)\,z) \\ \equiv\ & z\,(R\,z)\ . \end{aligned}$$

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