

MATHEMATICAL LOGIC — ASSIGNMENT ONE

(1) Prove that Peirce's Law $((p \supset q) \supset p) \supset p$ can be derived from the Law of Excluded Middle $p \vee \neg p$.

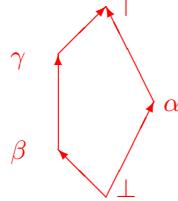
$$\begin{array}{c}
 \frac{[p]^3 \quad [\neg p]^1}{\frac{\perp}{\frac{\perp}{q}}} \neg E \\
 \frac{\perp}{q} \supset I^3 \\
 \frac{[(p \supset q) \supset p]^2 \quad p}{\frac{p \supset q}{p \vee \neg p}} \supset E^1 \\
 \frac{p \vee \neg p \text{ lem} \quad [p]^1}{\frac{p}{((p \supset q) \supset p) \supset p}} \vee E^2
 \end{array}$$

(2) Show that in any bounded distributive complemented lattice, each element has a unique complement.

This is Proposition 5.13 in the slides.

(3) Show an example of non-distributive lattice. Prove that the distributive law fails on it.

Consider the following order:



By direct inspection, this is a bounded lattice, since every pair of elements has a meet and a join, and \top and \perp are the maximum and the minimum of the order, respectively.

However,

$$\begin{aligned}
 \gamma \wedge (\beta \vee \alpha) &= \gamma \wedge \top = \gamma \\
 (\gamma \wedge \beta) \vee (\gamma \wedge \alpha) &= \beta \vee \perp = \beta
 \end{aligned}$$

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.