

## MATHEMATICAL LOGIC — ASSIGNMENT FOUR

- (1) Write a proof in natural deduction using the axioms of Peano arithmetic that every  $n \in \mathbb{N}$  is equal or greater than zero, where  $n \geq m$  is defined as  $\exists x. x + m = n$ .  
Discuss what happens if one defines  $n \geq m$  as  $\exists x. n = m + x$ .
- (2) Show that there is a non-standard model for Peano arithmetic.
- (3) Consider the Completeness Theorem for classical first order logic. Show that there is no proof of it which constructs a canonical model  $\mathfrak{M}$  for a theory  $T$  that is also classifying, i.e., such that, every other model of  $T$  can be obtained from  $\mathfrak{M}$  by a function which preserves truth.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

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*Date:* February 5<sup>th</sup>, 2019.