Mathematical Logic — Assignment Two

May 2nd, 2018

1. Prove that $\vdash A \wedge \exists x.\, B = \exists x.\, A \wedge B$ with $x \notin \mathrm{FV}(A)$. Find a counterexample when $x \in \mathrm{FV}(A)$.
2. Show that each ordinal S is totally well ordered by inclusion.
3. Consider the signature with addition, multiplication, 0, 1, and the $<$ relation, and let $\mathbb{R}$ be the real numbers with the standard interpretation of the symbols. Let T be the set of all true sentences (formulas with no free variables) in the structure $\mathbb{R}$ on that signature. Show that T has a model in which there is an infinite number, i.e. there is c for which $n < c$ for every $n \in \mathbb{N}$.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.