Mathematical Logic — Assignment Three

January 16th, 2017

1. Illustrate the Gödel-Gentzen translation and briefly discuss its meaning.

   See Definition 15.2, slide 347.

2. Prove that there is a combinator $\mathbf{Y}$ such that $\mathbf{Y}x =_\beta x(\mathbf{Y}x)$.

   See Theorem 17.2, slide 390.

3. Show that $\phi \equiv \neg\neg z \supset z$ cannot be proved in the intuitionistic propositional logic by providing a Heyting algebra which falsifies ϕ.

   Consider the Heyting algebra $0 < 1/2 < 1$, and interpret z as $1/2$. Since $\neg z$ is interpreted in 0, $\neg\neg z$ is interpreted in 1. Thus $\neg\neg z \supset z$ is true if and only if $1 \leq 1/2$, which is evidently false.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.